# Q2

Sets:

$V_1$: Set of potential facilities, indexed by $i$

$V_2$: Set of demand points, indexed by $j$

Parameters:

$d_j$: Demand of customer $j$

$c_{ij}$: unit transportation cost from facility $i$ to customer $j$

$f_i$: fixed cost of opening facility $i$

$q_i$: existing capacity of facility $i$

$q_i'$: maximum additional capacity at facility $i$

$p_i$: cost per unit of extra capacity expansion at facility $i$

$v_j$: penalty cost per unit of unmet demand at customer $j$

Decision Variables

$$y_i = \begin{cases} 1 & \text{if facility } i \text{ is opened} \\ 0 & \text{otherwise} \end{cases}$$

$x_{ij} \geq 0$ Quantity shipped from facility $i$ to demand point $j$

$s_j \geq 0$ amount of unmet demand at customer $j$

$z_i \geq 0$ additional capacity used at facility $i$

Objective Function

$$\min \quad Z = \underbrace{\sum_{i \in V_1} f_i y_i}_{\text{facility opening cost}} + \underbrace{\sum_{i \in V_1} \sum_{j \in V_2} c_{ij} x_{ij}}_{\text{transportation cost}} + \underbrace{\sum_{j \in V_2} v_j s_j}_{\text{unmet demand penalty}} + \underbrace{\sum_{i \in V_1} p_i z_i}_{\text{capacity expansion cost}}$$

$$\sum_{i \in V_1} x_{ij} + s_j = d_j \qquad \forall j \in V_2 \qquad \rightarrow \text{demand satisfaction}$$

$$\sum_{j \in V_2} x_{ij} \leq q_i y_i + z_i \qquad \forall i \in V_1 \qquad \rightarrow \text{facility capacity constraint}$$

$$0 \leq z_i \leq q_i' y_i \qquad \forall i \in V_1 \qquad \rightarrow \text{capacity expansion limit}$$

$$x_{ij} \geq 0, \quad s_j \geq 0, \quad z_i \geq 0 \qquad \forall i, j$$

$$y_i \in \{0, 1\} \qquad \forall i$$